11.5 Q 10.

a) The differential equation says that the rate ~~of~~ ~~&~~ cooling at which the coffee cools is proportional to the difference between the temperature of the ~~sou~~ surroundings and that of the coffee.

Since dT/dt is negative — the coffee is cooling and

$(T-20)$ is positive; the coffee is warmer than room temperature

k is positive

b)

$$\frac{dT}{dt} = -k(T-20)$$

$$\int \frac{dT}{(T-20)} = \int -k\,dt.$$

$$\ln|T-20| = -kt + c.$$

$$T - 20 = Ae^{-kt} \quad ; \quad A = e^{c}.$$

and

$$T(t) = 20 + Ae^{-kt}.$$

Since coffe is made to boil its temperature must be $100^\circ C$ and

Substituting for $k = \frac{1}{2}\ln 8/7$

$$t_{60} = \frac{1}{\frac{1}{2}\ln 8/7} \cdot \ln 2 \approx 10 \text{ mins.}$$

Comments!

Most of you did not see that boiling coffee would be at 100°C and that is needed to obtain $A = 80°C$.

Also the few ~~wat~~ who got the question right failed to see that the value for t_{60} is an approximation and not exact.

$A = 80°c.$

$T(t) = 20 + 80e^{-kt}$. — ①

Given $t = 2$mins as time taken for coffee to cool to 90°C.

$$90 = 20 + 80e^{-k(2)}$$

$$70 = 80e^{-2k}$$

$$e^{2k} = \cancel{7/8}\ 8/7.$$

$$2k = \ln 8/7$$

$$k = \tfrac{1}{2}\ln 8/7.$$

if t_{60} is the time taken for coffe to cool down. to 60°C.

then from ① we can write:

$$60 = 20 + 80e^{-kt_{60}}$$

$$\cancel{40}^{1} = \cancel{80}^{2}e^{-kt_{60}}$$

$$1/2 = e^{-kt_{60}}$$

$$kt_{60} = \ln 2$$

$$t_{60} = \frac{1}{k}\ln 2$$

11.6 Q12

Since the rate at which the volume, V, of the moth ball is decreasing, is proportional to ~~the~~ its surface area A, we write the differential equation:

$$\frac{dV}{dt} = -kA.$$

we obtain dV/dt from the formula of the volume of a sphere with radius r.

$$V = \frac{4}{3}\pi r^3 \qquad \Rightarrow \qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

also the area of a sphere is given by $A = 4\pi r^2$.

hence

$$\cancel{4\pi r^2}\, dr/dt = -k\, \cancel{4\pi r^2}$$

$$dr/dt = -k.$$

we are given that the radius of the moth ball reduces from 1 cm to 0.5 cm after 1 month.

$$\Rightarrow \frac{1 - 0.5 \text{ cm}}{\cancel{0}1 - 0} = 0.5 \text{ cm/month} = k.$$

$$\frac{dr}{dt} = -0.5$$

and

$$r = -0.5t + r_0.$$

but when $t = 0$, $r = 1 = r_0$

$$r = -0.5t + 1$$

we then find t when $r = 0.2$ cm ; t is the time taken for the mothball to reduce to 0.2 cm

$$0.2 = -0.5t + 1$$

$$0.5t = 1 - 0.2$$

$$t = 0.8/0.5$$

$$t = 1.6 \text{ months}$$

Comments:

Most students got this question wrong and the problem was setting up the right differential equation.